"An expert is a person who has made all the mistakes that can be made in a very narrow field."
Niels Bohr

4 Three-fluid model

Three-fluid two-phase model without evaporation or condensation:

- Mass conservation
- Momentum conservation
- Energy conservation
- Pressure equation
- Fluid properties on a general form

4.1 General

Let us now go one step further than the previous chapter in adding complexity. We still have only two fluids, but in annular flow, liquid can occur both as a film around the pipe wall and as droplets carried by the gas core. The one gas and two liquid forms can be described by three different mass, momentum, and energy conservation equations and we refer to such a formulation as a *three-fluid model*.

Annular flow is one of the most common flow patterns encountered in natural gas well-bores and pipelines. It occurs at high gas and low to medium liquid flow-rate and at all pipe elevation angles.

Droplets can be torn from liquid film (droplet entrainment), and droplets can also settle and become part of the film again (liquid film deposition). Depending on the entrainment and deposition rates, it is known that in the most extreme cases, all the liquid can flow as liquid film or (nearly all) as droplets.

In this model, we do not assume the flow to necessarily be isothermal. Apart from that, we keep the main simplifications from chapter 3 (no boiling or condensation, the pipe does not have perforations, the flow regime does not change).

As in chapter 3, we denote quantities referring to the gas with a G-subscript. Continuous liquid is given L as a subscript, and liquid in droplet form has subscript D. We will show how to establish all necessary conservation equations, correlations for friction, droplet entrainment and liquid film deposition for such a three-fluid model.

The model in this chapter is general and would be valid for other flow regimes than annular if we set the droplet fraction $\alpha_D = 0$, but we are not going to focus on anything other than annular flow.

4.2 Mass conservation

We can now write 3 continuity equations based on equation 2.2.4. For the gas phase, we get:

$$\frac{\partial(\alpha_G \rho_G)}{\partial t} = -\frac{\partial(\alpha_G \rho_G v_G)}{\partial x} + \Gamma_{LG} + \Gamma_{DG} + \Gamma_{GW} \qquad (4.2.1)$$

Since we are dealing with two different fluids which do not change phase, $\Gamma_{LG} = \Gamma_{DG} = 0$. Also, no gas is going to be added through perforations in the pipe wall, and therefore $\Gamma_{GW} = 0$. Equation 4.2.1 becomes:

$$\frac{\partial(\alpha_G \rho_G)}{\partial t} + \frac{\partial(\alpha_G \rho_G v_G)}{\partial x} = 0 \qquad (4.2.2)$$

Liquid can jump between liquid film and droplets, though, in a process called liquid entrainment and droplet deposition. For the liquid film we get:

$$\frac{\partial(\alpha_L\rho_L)}{\partial t}+\frac{\partial(\alpha_L\rho_L v_L)}{\partial x}=-\Gamma_{LDi}+\Gamma_{DLi} \quad (4.2.3)$$

The droplets are assumed to have the same density as the liquid film. Using equation 2.2.4 and 2.2.5, we get:

$$\frac{\partial(\alpha_D\rho_D)}{\partial t}+\frac{\partial(\alpha_D\rho_L v_D)}{\partial x}=+\Gamma_{LDi}-\Gamma_{DLi} \quad (4.2.4)$$

Equation 2.2.6 becomes:

$$\alpha_G+\alpha_L+\alpha_D=1 \quad (4.2.5)$$

4.3 Momentum conservation

Equation 2.3.16 for the gas yields:

$$\frac{\partial(\alpha_G\rho_G v_G)}{\partial t}=-\frac{\partial(\alpha_G\rho_G {v_G}^2)}{\partial x}-\alpha_G\frac{\partial p_G}{\partial x}+R_{LGi}+R_{DG}+R_{Gw}$$
$$+S_{LG}+S_{DG}+S_{Gw}-\alpha_G\rho_G g\sin\theta \quad (4.3.1)$$

R_{LG} represents friction force from liquid film on the gas, R_{DG} is friction force pr. unit pipe volume from the droplets on the gas, and R_{GW} is similarly volume-specific friction force from the wall on the gas. In our model we presume the film covers the entire wall's surface. Therefore, there is no direct contact between gas and pipe wall, and $R_{GW}=0$.

If all surface tension forces acting directly on the gas flow are negligible, we can set $S_{LGi} = S_{DGi} = S_{Gw} = 0$. For simplicity, we also assume the pressure to be constant across all phases in each cross-section so $p = p_G = p_L = p_D$ (even though we know that can lead to the equations not being hyperbolic):

$$\frac{\partial(\alpha_G \rho_G v_G)}{\partial t} + \frac{\partial(\alpha_G \rho_G {v_G}^2)}{\partial x} = -\alpha_G \frac{\partial p}{\partial x} - R_{GL} - R_{GD} - \alpha_G \rho_G g \sin\theta \tag{4.3.2}$$

Similar momentum equation for the liquid film:

$$\frac{\partial(\alpha_L \rho_L v_L)}{\partial t} + \frac{\partial(\alpha_L \rho_L {v_L}^2)}{\partial x} = -\alpha_L \frac{\partial p}{\partial x} - v_{L(D)}\Gamma_{LD} + v_{D(L)}\Gamma_{DL} + R_{GL} + R_{DL} - R_{Lw} - \alpha_L \rho_L g \sin\theta \tag{4.3.3}$$

We have adopted the notation $v_{L(D)}$ for the average velocity of liquid becoming entrained as droplets. This can be approximated as the average liquid film velocity v_L (as we will do in the example in chapter 5). But the absolute velocity $v_{L(D)}$ should not be confused with v_{LD}, a notation used for velocity difference between liquid film and droplets – a parameter relevant to some of the friction calculations.

For the droplets:

$$\frac{\partial(\alpha_D \rho_L v_D)}{\partial t} + \frac{\partial(\alpha_D \rho_L {v_D}^2)}{\partial x} = -\alpha_D \frac{\partial p}{\partial x} + v_{L(D)}\Gamma_{LD} - v_{D(L)}\Gamma_{DL} - R_{DL} + R_{GD} - \alpha_D \rho_L g \sin\theta \tag{4.3.4}$$

4.4 Energy equation

By summarizing equation 2.4.4 for all phases, and applying equations 2.4.5 - 2.4.9, we get:

$$\frac{\partial}{\partial t}(\alpha_G E_G + \alpha_L E_L + \alpha_D E_D) + \frac{\partial}{\partial x}[\alpha_G v_G (E_G + p) + \alpha_L v_L (E_L + p) + \alpha_D v_D (E_D + p)] = \Gamma_{Gw} h_{Gw} + \Gamma_{Lw} h_{Lw} + \Gamma_{Dw} h_{Dw} + q + w \tag{4.4.1}$$

Enthalpy from mass sources (contained in any fluid flowing in through the pipe wall) is assumed to come in the form of gas, liquid film, or droplets. q is volume-specific heat from the environment through the pipe wall into the fluid, and w is work carried out on the fluid (in pumps or compressors, or negative work in a turbine).

4.5 Fluid properties

Since this model presumes no gas will become liquid or vice versa, gas and liquid properties can be considered independent of each other. The main properties are simply the state equations correlating pressure, temperature and density for the gas and liquid separately, as they were shown in equations 3.7.1 and 3.7.2.

The viscosities are of course involved in the friction calculations, and like the densities, they generally depend both on pressure and temperature:

$$\mu_G = \mu_G(p, T) \tag{4.5.1}$$

$$\mu_L = \mu_L(p, T) \tag{4.5.2}$$

The liquid's surface tension when in contact with the particular gas is also involved, so we also need:

$$\sigma_{LG} = \sigma_{LG}(p, T) \tag{4.5.3}$$